Effects Of Ranging In Circular Polarization

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Abstract

As reported at the 2007 Grasse Technical Workshop, Stromlo SLR can alternate between linear and circular polarization transmitted beams by the insertion and removal of a quarter-wave plate (QWP). Control of insertion and removal allows coincidence with Normal Point bin boundaries. This paper presents an analysis of data obtained using ETS-8 during a very clear night in September 2008, in addition to a re-analysis of the 4 Lageos passes described in the Grasse (2007) workshop. By careful use of classical statistics, the results show significant differences between the two polarization states in range measurements and return rates, in the senses predicted by Dave Arnold. Correlations depending upon the angle between the transmitted (linear) polarization vector and the velocity aberration vector are also examined. The targets used are suitable for this experiment because their retroreflectors are uncoated.

Introduction

It has been predicted by Arnold (2002) that, when ranging to spherical targets having uncoated retroreflectors, the use of circular polarization will result in:

- Shorter ranges, by about 4 mm in the Lageos cases,
- Greater return rates,
- Reduced scatter.

In particular, he states:

> "If linear polarization is used, the transfer function with uncoated cubes has a "dumbbell" shape which can introduce a systematic error if no correction is applied. The problem can be corrected by applying a correction for the asymmetry. The asymmetry can be eliminated by using circular polarization."

and

> "The Lageos 2 retroreflector array was tested in the laboratory before launch . . . the testing showed a difference in the range correction for Lageos between linear and circular polarization".

Arnold also predicts that, in linear polarization, they will be affected by the angle between the satellite's velocity aberration vector and the direction of the polarization vector as it reaches the target, which here will be called the "Arnold angle". This is a candidate explanation for the anecdotal observation by several observers at different stations that the satellite "seems to disappear in certain parts of the sky", although another possible explanation is that the turning mirrors in the telescope's Coude transmit path have unequal p- and s-reflectances, due perhaps to coating degradation.

Lageos-1 and -2 were the prime targets in this study. ETS-8 was also chosen because, although its cube-corner array is planar, it carries uncoated cubes. Being in geostationary orbit, the array geometry as seen by a ground station is nominally constant.

During these studies, anomalies showed up in the calculations of velocities from the CPF predictions of satellite position.

Ranging Differences Between Circular And Linear Polarization

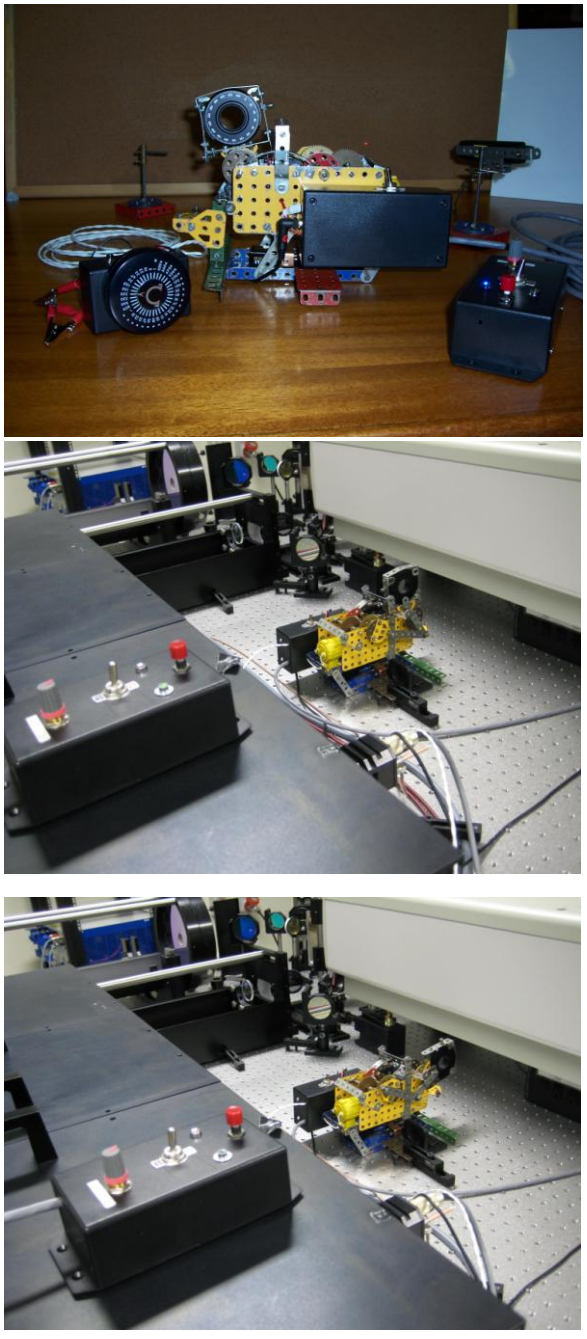

Figure 1. Quarter Wave-Plate Inserter.
(Top) In fabrication workshop.
(Middle) On laser table, QWP OUT of path.
(Lower) On laser table, QWP IN laser path.

Experimental Setup and Processing Strategy

A quarter wave plate (QWP) was mounted in an Inserter to convert the outgoing laser from linear to circular polarization. The Inserter (Figure 1) was fabricated from a mechanical engineering prototype system, and placed on the laser table between the frequency doubler and the Transmit/ Receive mirror, so it did not affect the return path. Alternate insertion and withdrawal of the QWP into the laser beam was performed by remote control. The laser was disabled for a few seconds during these changes of state, which generally occurred on Normal Point bin boundaries. Thus, alternate Normal Points were in either the IN (circular) or OUT (linear) state, so minimizing variations due to atmospherics, the Arnold angle, poor trend-curve fitting during processing, and so on.

Full-rate data files from normal Stromlo post-processing were used. They contained only returns accepted by the final filter. CPF predictions from HTSI (JAXA for ETS-8) were interpolated iteratively to “bounce” time at the satellite on ITRF X,Y,Z coordinates using an 8-point Lagrange interpolator, before calculating the topocentric ranges. Atmospheric refractions corrections were applied using the Mendes-Pavlis formula (ILRS RSG, 2002-4).

The predictions were completed by fitting polynomials of degree 1-9 through the residuals so formed, including both IN and OUT data together, and the lowest-degree adequate fit was chosen subjectively. (It was felt that fitting all returns together gave a better common baseline for comparing INs vs. OUTs rather than

fitting separate curves.) Evaluations of the selected polynomial at each return were added to the earlier predictions, and new residuals (here called "***departures***") were formed, from which Normal Points were calculated in accordance with ILRS instructions (Sinclair, 1997). The results of a Lageos-1 pass re-processed in this manner, shown in Figure 2, suggest that the trend curve adopted does indeed remove all systematic variations. It is noted that the Normal Points produced by this special processing are, in general, somewhat different from the regular Stromlo NPs, possibly due to different choices of trend functions; in this study, strenuous efforts were made to get the "departures" graph as flat and smooth as possible.

Observations

Passes observed with the QWP operation were:

Lageos-1, 2007 Sep 08 at 12:55 UTC (night)
Lageos-1, 2007 Sep 09 at 15:05 UTC (night)
Lageos-2, 2007 Sep 11 at 23:20 UTC (day)
Lageos-2, 2007 Sep 12 at 07:40 UTC (evening)
ETS-VIII, 2008 Sep 11 at 09:47 UTC (night) (the next year)

and, without QWP operation as a "contra" check on interpretation of the analyses:

Lageos-2, 2008 Sep 10 at 12:30 UTC (night).

Some exploratory results on the first four Lageos passes were presented at the Grasse ILRS Technical Workshop (Luck et al, 2007).

Statistical Analysis

For the comparisons between circular (C) and linear (L) polarizations, only returns lying in "Adjacent Pairs" of NPs were used, i.e. in bins having returns from the other state in at least one adjoining bin. This restriction sought to reduce further any observational bias between states, and was applied to both NP and FR comparisons.

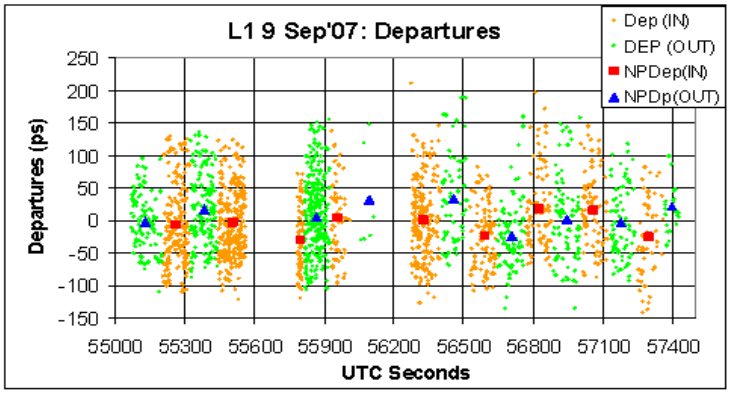

Figure 2. Residuals from final trend curve ("departures") for a Lageos-1 pass, showing also the Normal Points. The trend curve was a polynomial of degree 6 (order 7).

The tests performed, and reported in Table I, were based on elementary statistical hypothesis testing (see e.g. Hoel (1966)) and the statistical tables contained therein (and elsewhere). The tests are all one-tailed and assessed in terms of percentage confidence that the null hypothesis has been rejected correctly, i.e. using the *'p-value'*, e.g.:

Confidence = 100 [1 – Pr{ $z > \hat{z}$ | null hypothesis (H_0) is true }]

or equivalent statement for other tests, and $\hat{z}$ is the calculated test statistic appropriate to the test.

Difference between Means

We define Student's t statistic as,:

$\hat{t}$ = [(Mean of NP/ FR departures, C) – 3.1 ps – (Mean of NP/FR departures, L)]/ s

where the extra delay due to QWP presence in the transmit path is 3.1 ps (catalog data), except for the "contra" pass, and, for example $s = \sqrt{s_C^2 / n_C + s_L^2 / n_L}$ is the "pooled" sample

standard error of the difference. Here n_C, n_L and s_C^2, s_L^2 are the numbers of NPs (or FR points) and the variances of the NPs (or FR points) about their means in each state. There are $n_C + n_L - 2$ degrees of freedom (d.f.).

Ratio of Normal Point Variances

Fisher's F test statistic is given by $\hat{F} = s_L^2 / s_C^2$ with $n_L - 1, n_C - 1$ degrees of freedom. Note that, if Arnold's prediction is true that C-scatter is less than L-scatter then $\hat{F}$ is significantly greater than 1. But there is little statistical power in this test for FR data.

Return Rates

The 'population proportion' test was used: in each adjacent pair, the ratio of return rates was calculated, viz: $r_i = [returnrate/shot(C)]/[returnrate/shot(L)]$, from which the proportion $\hat{p} = [num.pairs \text{ with } r_i > 1]/[total\ num.pairs]$ was obtained. If return rates are equal, then $H0: p_0 = 0.5$ is expected, and tested against $H1: p > 0.5$, with the test statistic $\hat{z} = (\hat{p} - p_0)/\sqrt{p_0(1-p_0)/n}$ being distributed as Normal(0,1). For the passes in this experiment it was difficult to get the number of shots fired per bin, so the time interval between the first and last accepted returns in a bin was used instead.

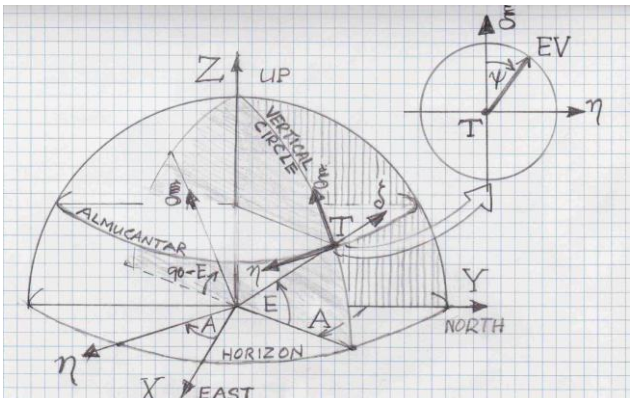

Figure 3. Position angle ψ in local ENU coordinates.

Discussion of Results

The results given in Table 1 come from a re-computation after the Poznan Workshop, and are less optimistic that ranging in circular polarization produces measurable improvements. Greater attention was paid to the boundaries between QWP 'IN' and 'OUT' states. Though the changes were generally small, they sometimes caused quite pronounced differences. We conclude that this statistical instability means that the data are not yet sufficiently precise to draw firm conclusions. Nevertheless, some of the results were suggestive:

- The return rate proportions (see row "Confidence: C rate>L rate") seem to suggest higher rates with circular polarization. However, the test used is not particularly sensitive so a better one is sought.
- With FR data, the mean difference in ranges was highly significant (see rows "Confidence: C mean<L mean"), except for one QWP pass and, of course, the check pass. However, with NP data, none of the passes yielded significant (>95%) differences in the means.
- Neither the FR nor the NP tests showed significant differences in variances (see rows "Confidence: C RMS<L RMS").

Table 1. Results of Statistical Tests

C = circular polarization ("IN"), L = linear polarization ("OUT"). Data from Adjacent Pairs only.

Target	LAG-1	LAG-1	LAG-2	LAG-2	ETS-8	LAG-2 *
Date observed	8 Sep'07	9 Sep'07	11Sep'07	12Sep'07	11Sep'08	10Sep'08
FULL RATE DATA						
Num.returns (C)	1813	1040	1397	1958	1372	672
Num.returns (L)	1292	813	1107	2092	1064	646
Mean departure (ps) (C)	0.6	-2.4	-1.6	-1.0	-2.0	3.0
Mean departure (ps) (L)	-0.7	5.6	2.6	-0.3	0.4	-3.0
Mean(C)– 3.1 –Mean(L) (ps)	-1.8	-11.1	-7.3	-3.8	-5.5	* 6.0
SS RMS (ps) (C)	66.4	57.9	53.0	47.6	47.4	43.1
SS RMS (ps) (L)	70.7	58.0	50.8	48.3	48.0	40.5
Pooled RMS (ps)	2.5	2.7	2.1	1.5	2.0	2.3
Student's 't' of Diff.of Means	-0.718	-4.113	-3.513	-2.549	-2.819	2.608
Confidence: C mean<L mean	**76.4%**	**100.0%**	**100.0%**	**99.5%**	**99.8%**	**< 50 %**
Fisher's 'F' of Variance Ratio	1.13	1.00	0.92	1.03	1.03	0.88
Confidence: C RMS < L RMS	**99.2%**	**50.7%**	**6.7%**	**73.2%**	**67.3%**	**5.1%**
NORMAL POINTS						
Num.Normal Pts, =(C) = (L)	9	8	11	12	13	8
Mean NP departure (ps) (C)	0.1	-2.5	-4.4	-1.9	-1.6	1.7
Mean NP departure (ps) (L)	0.2	6.8	2.8	-1.2	1.4	-3.2
Mean(C)- 3.1- Mean(L) (ps)	-3.2	-12.4	-10.3	-3.8	-6.1	* 4.9
RMS of NPs (ps) (C)	12.6	15.8	11.1	14.3	13.7	12.2
RMS of NPs (ps) (L)	22.2	17.5	16.5	17.5	11.6	11.0
Pooled NP RMS (ps)	8.5	8.3	6.0	6.5	5.0	5.8
Students't' of Diff of NP Means	-0.373	-1.489	-1.716	-0.575	-1.229	0.841
Confidence: C-mean<L-mean	**64.3%**	**92.1%**	**94.9%**	**71.4%**	**88.5%**	**< 50%**
Fisher's 'F' of Variance Ratio	3.11	1.24	2.21	1.50	0.71	0.82
Confidence: C RMS < L RMS	**93.5%**	**60.7%**	**88.6%**	**74.4%**	**28.1%**	**39.8%**
RETURN RATE PROPORTION						
Num. Adjacent Pairs	9	8	11	12	13	8
Num.APs with C rate > L rate	9	7	9	8	5	6
Normal 'z'	3.00	2.12	2.11	1.16	-0.83	1.41
Confidence: C rate > L rate	**99.9%**	**98.3%**	**98.3%**	**87.6%**	**20.3%**	**92.1%**

Effects In Linear Polarization

Electric Vector Orientation

Depending on the type of coatings on the mirrors in the transmit optical path and their condition, there may be changes in the direction and intensity of the electric vector (EV) if transmitting in linear polarization. These changes are due to inequalities of the p- (parallel to plane of incidence) and s- (perpendicular) components of the reflectances. They produce variations as the Coudé mirrors rotate in azimuth and elevation because the planes of incidence change even though the angles of incidence remain constant at 45^{o}. If the p- and s-reflectances are substantially unequal, this phenomenon could be a candidate as the cause of disappearing returns in certain sections of otherwise crystal-clear skies.

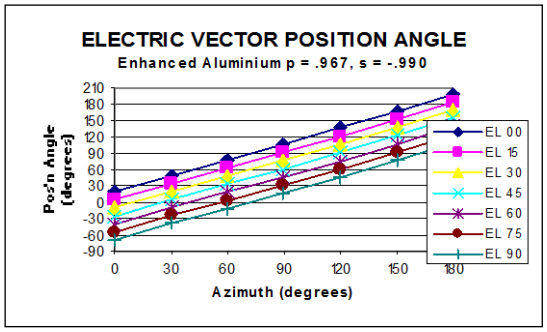

Figure 4. Difference in EV position angle between enhanced aluminium and perfect coatings, as telescope rotates.

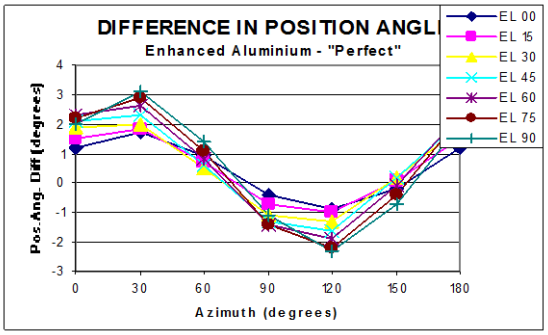

Figure 5. General behaviour of EV position angle as telescope rotates in azimuth and elevation.

For Figures 4-6, the EV was mapped through the Stromlo SLR as if all the 45° turning mirrors after the laser table were coated with enhanced aluminium, p = 0.967, s = 0.990. In effect, four surfaces were experiencing 'plane-of-incidence' changes. Curved surfaces were ignored. The initial EV is vertical, while the final EV is given in terms of its "position angle" with respect to the vertical circle (see Figure.3). Comparisons were made against "perfect" coatings (p = s = 1.00). Figure 4 shows the overall variation of the EV position angle as the telescope rotates in azimuth and elevation. Figure 5, at much larger scale, demonstrates the variations amounting to about 4° due to unequal reflectances, compared with perfect coatings. Figure 6 reveals a 14% variation in final transmitted energy (square of EV amplitude), which is quite appreciable even for reflectances relatively close to 1.

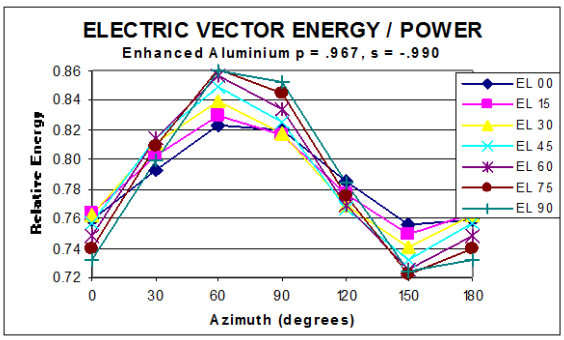

Figure 6: Electric vector energy as telescope rotates, with enhanced aluminium coatings.

An initial experiment to measure transmitted energy at Herstmonceux SLR in 2007 showed somewhat similar variations (8%) as their telescope rotated in azimuth only, possibly indicating coatings degradation (Smith and Appleby, 2007).

Arnold Angle

Given the EV position angle algorithm, it was relatively easy to calculate the "Arnold Angle", i.e. the angle, at the target, between the incident polarization vector and the velocity aberration vector which is the relative inertial velocity of the satellite w.r.t. the station (Arnold, 2007). This angle is virtually constant at 122.8° for ETS-8 tracked from Stromlo, as expected, since ETS-8 is in geostationary orbit. The result for one of the Lageos test passes is shown in Figure7. However, it is not thought feasible at present to detect this effect in view of all

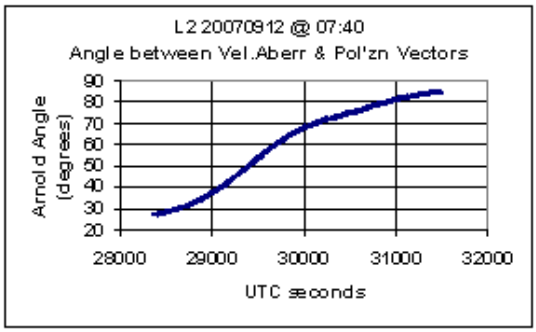

Figure 7. Angle between polarization vector and velocity aberration vector, for a Lageos pass tracked from Stromlo.

the other factors affecting return rates, especially as Stromlo does not have a return signal strength monitor.

Trend Function

In studies of this sort it is crucial that the trend function from which departures are formed be able to remove all known and unknown systematic errors in the data. For this, fitting simple models such as range- and time-bias ($residual_i = rangebias + rdot_i*timebias$) (which have physical meaning) prior to empirical polynomial fitting gave inconsistent results, as did fitting a set of osculating Kepler elements. These depend on the $rdot_i$ which were obtained by numerical differentiation of CPF position predictions using an 8-point Lagrange differentiator rigorously tested against known functions and Werner Gurtner's HERMITE subroutine.

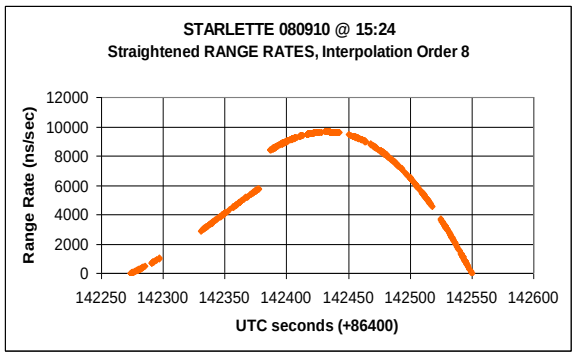

Figure 8. Range rates obtained by numerical differentiation of CPF positions. A trend line has been removed to highlight discontinuities.

Upon investigation, discontinuities were found in the rates so calculated (Fig.8). A study is continuing to ascertain whether the problem lies with my software (inconceivable!) or with the CPF files. This might also explain a quite-often-observed difficulty obtaining nice flat "departures" in regular Stromlo processing.

Summary and Conclusions

From judicious elementary statistical analysis of the limited data collected on satellites carrying uncoated cube corners, there is tantalizing evidence suggesting that:

- Range measurements are shorter in circular polarization than in linear, when comparing Full-Rate data though not when comparing Normal Points. This applies to the planar-array ETS-8 as well as to the spherical-array Lageos;
- Return rates are perhaps greater in circular polarization than in linear, but only for the spherical-array Lageos targets;
- There is no evidence of differences in RMS scatter between polarizations;
- The Lageos "check" pass showed no statistically significant differences, as expected since there were none.

However, these effects are considered to be merely on the verge of detectability under the conditions obtaining at the time of the Stromlo experiments, so at this stage we make no firm recommendation on the desirability of ranging in circular polarization.

When ranging in linear polarization, there will theoretically be appreciable variations in transmitted electric vector orientation and especially energy as the telescope rotates in azimuth and elevations, if the p- and s-reflectances of the Coudé mirrors are unequal. There is perhaps some supporting observational evidence from Herstmonceux. An algorithm has been developed for interpreting effects of the "Arnold angle" if ever they become detectable. Worrying discontinuities in CPF predictions are suspected.

Acknowledgements

Vicki Smith took the Herstmonceux energy measurements. Bart Clarke (HTSI) has provided CPF files with velocity data for further investigation of CPF anomalies.

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